ELECTRICAL BREAKDOWN OF CONDUCTORS. FUSION STAGE

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The physical processes accompanying electrical explosion of conductors are varied in nature; solid–liquid metal and liquid–vapor phase transitions, stratum formation, vapor breakdown, arc discharge ignition, etc. [1-6]. At the present time there exists no complete description of this process, even of a qualitative nature. In such a situation it is natural to turn to computer simulation of physical models of the various stages of breakdown, with subsequent verification of results in goal-directed experiments. The present study is dedicated to the initial stage of the breakdown process – the fusion stage. Not much attention has been directed to this stage in experimental and numerical studies of an applied nature, which is natural to a certain degree, since fusion is energetically a less significant process than, e.g., boiling. However, for the justification of theoretical positions (in particular, we will consider a two-phase solid metal–liquid model), study of the fusion stage is of significant interest.

The objects studied were in the form of foil. The fusion rate and resistance of a foil as a function of the energy applied to it are dependent on the ratio of the characteristic times

$$\tau_1 = h_0^2/\chi, \quad \tau_2 = \lambda n \sigma_0 / j_0^2,$$

where $2h_0$ is the foil thickness; λ, specific heat of fusion; χ, thermal diffusivity; j_0, initial current density; σ_0, conductivity; n, density of the material; τ_1, characteristic temperature relaxation time; and τ_2, parameter characterizing the rate at which energy is introduced and the time for foil melting.

We will consider the following melting regimes, possible for impulse heating of a foil by currents of various densities, and characterized by different values of the parameter $\delta^2 = \tau_1/\tau_2$ [7].

In the case $\tau_1 \ll \tau_2$ ("slow" introduction of energy into the foil) the temperature can be considered constant over the entire section – all the energy dissipated goes into fusion. We write an expression for the foil resistance assuming that melting occurs from the specimen surface [8] and that the melting front is planar

$$R = \frac{l}{2b} \frac{1}{\sigma_1 h(t) + \sigma_2 [h_0 - h(t)]}, \tag{1}$$

where l and b are the length and width of the foil; h(t), thickness of the solid phase; σ_1, σ_2, conductivities of the solid and liquid phases. In as much as the thermal conductivity is high, the thermal balance condition has the form

$$E = 2\lambda n l b [h_0 - h(t)]. \tag{2}$$

Combining Eqs. (1) and (2), we obtain

$$R(E) = \frac{R_0}{1 + \frac{E}{Q}\left(\frac{\sigma_2}{\sigma_1} - 1\right)}, \tag{3}$$

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where $Q = 2\lambda n h_0 l b$, $R_0 = l/(2h_0 b\sigma_1)$.

For the case $\tau_1 > \tau_2$ ("rapid" energy introduction) all the energy E introduced into the foil cannot reach the melting front because of thermal conductivity. Thus, a portion of the energy is lost to superheating of layers of the solid and liquid phases far from the melting front. The fraction of energy expended in such heating is larger, the larger the ratio τ_1/τ_2. Using the thermal conductivity equation one can, as in [7], obtain an estimate of the superheating in the center of the foil for $\delta^2 \gg 1$

$$\frac{\delta T}{T_f} \sim \frac{j_0^2}{T_f\, n c_p \sigma_0 \tau_2} \sim \frac{\lambda}{c_p}\frac{1}{T_f} \sim 1,$$

where T_f is the fusion temperature under normal conditions; c_p is the specific heat. To determine R(E) in this case it is necessary to know the temperature distribution over the foil section.

Formulation of the Problem of Temperature Profile over Foil Section. Neglecting heat exchange with the surrounding medium and boundary effects, we write the thermal conductivity equations for the solid (index 1) and liquid (index 2) phases

$$\frac{\partial T_i}{\partial t} = \chi \frac{\partial^2 T_i}{\partial x^2} + \frac{j_i^2}{n_i c_{pi} \sigma_i(T)} \qquad (i = 1, 2) \tag{4}$$

(the x axis is directed perpendicular to the foil surface, and the origin is at the foil center (Fig. 1)). The unknown T_i and the thickness h(t) satisfy the thermal balance condition on the phase boundary

$$n_2 \lambda \frac{dh}{dt} = \varkappa_1 \frac{\partial T_1}{\partial x}\Big|_{x=h(t)} - \varkappa_2 \frac{\partial T_2}{\partial x}\Big|_{x=h(t)}, \tag{5}$$

where $\varkappa_i$ is the thermal conductivity and c_{pi} is the specific heat.

We assume that the quantities $n_1 = n_2$, $c_{p1} = c_{p2}$, $\chi_1 = 2\chi_2$ are independent of temperature and that the total current through the foil is constant

$$I = I_1 + I_2 = \mathrm{const}.$$

The temperature dependence of the conductivity may be expressed in the form

$$\sigma_i(T) = \frac{\sigma_{0i}}{1 + \beta_i (T_i - T_f)}, \tag{6}$$

where σ_{0i} is the conductivity at $T = T_f$; β_i is the resistance temperature coefficient.

We write the current density in the form

$$j_i = \frac{j_{0i}}{\sigma_{0i}} \sigma_i = \frac{j_{0i}}{1 + \beta_i (T_i - T_f)}. \tag{7}$$

The value of the initial current density per unit foil width may be expressed in terms of the total current and the initial conductivity

$$j_{0i} = I \frac{\sigma_{0i}}{k_1 + k_2}, \quad k_1 = \int_0^h \sigma_1 dx, \quad k_2 = \int_h^{h_0} \sigma_2 dx.$$

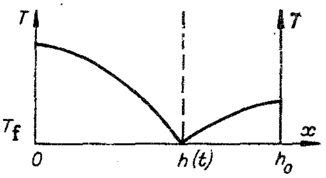

Fig. 1

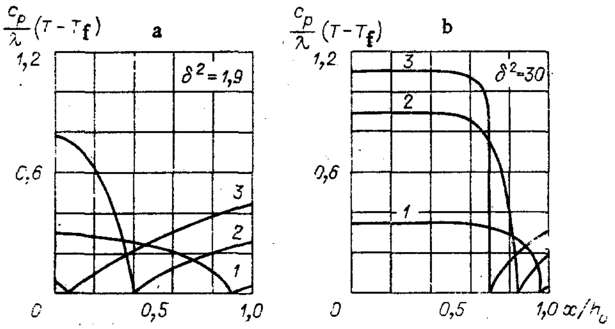

Fig. 2

Substituting Eqs. (60, (7) in Eq. (4) and using Eq. (5), we obtain a system of equations

$$\frac{\partial T_i}{\partial t} = \chi \frac{\partial^2 T_i}{\partial x^2} + \frac{j_{0i}^2}{nc_p \sigma_{0i}} \frac{1}{1+\beta_i (T_i - T_f)},$$

$$n\lambda \frac{dh}{dt} = \varkappa_1 \frac{\partial T_1}{\partial x}\bigg|_{x=h(t)} - \varkappa_2 \frac{\partial T_2}{\partial x}\bigg|_{x=h(t)} \tag{8}$$

with boundary conditions:

$$\frac{\partial T_2}{\partial x}\bigg|_{x=h_0} = 0 \quad \text{(heat exchange with surrounding medium absent)},$$

$$\frac{\partial T_1}{\partial x}\bigg|_{x=0} = 0 \quad \text{(from symmetry considerations)},$$

$$T_1|_{x=h} = T_2|_{x=h} = T_f.$$

System (8) was solved numerically for various parameter values with the aid of an implicit difference scheme of the Crank–Nicholson type [9] to second-order accuracy in both variables.

Results of Numerical Solution. Figure 2a, b (curves 1-3 correspond to times $t = 0.3\ \tau_2$; $0.75\tau_2$; $0.9 \cdot \tau_2$) illustrate the dynamics of the temperature profiles in the solid and liquid phases during melting. As follows from Fig. 2a, the temperature on the outer boundary of the liquid phase increases monotonically. In the center of the solid phase, the temperature also increases at first, but then after reaching some maximum value, it falls rapidly. This can be explained by the fact that at $\delta^2 > 1$ and $h(t) \approx h_0$ the major portion of the heat liberated is expended in increasing the temperature (the material at the center does not "perceive" the effect of the phase transition). In later stages when the thickness of the solid metal layer has decreased to $h(t) \approx (\tau_1\chi)^{1/2}$, an effective removal of heat to the phase transformation zone develops because of thermal conductivity.

It is interesting that a higher temperature gradient exists in the solid metal layer at the start of the process: This indicates that transfer from the solid phase to the fusion boundary is more effective, and in a first approximation it is this fact which determines the rate of displacement of the phase boundary.

Figure 2b shows results for rapid introduction of heat ($\delta^2 = 30$). Even in the initial stage of the process the temperature distribution over foil section takes on its characteristic П-shaped profile (the temperature in the solid phase is practically constant).

The results of numerical caluclation of the temperature profile (using Eq. (6)) can be represented in integral form as the dependence of total foil resistance upon energy introduced into the foil $R/R_0 = f(E/Q)$. The functions obtained in this manner from two quite different values of the parameter $\delta^2 = 1.9$ and 30 were compared with experimental values.

Experiments and Evaluation of Results. The experimental method and measurement technique were similar to those of [10].

There were certain difficulties in achieving the total current constancy assumed in the calculations. Using certain approximations it was possible to produce regimes in which this requirement was fulfilled satisfactorily. The oscillograms of $I(t)$ and $U_R(t)$, shown in Fig. 3, illustrate fulfillment of this condition, with foil melting occurring when the current reaches its maximum value, where $I = \text{const}$.

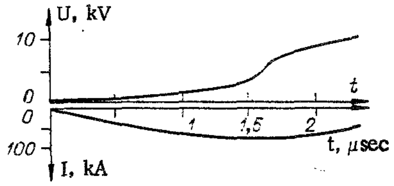

Fig. 3

In accordance with these estimates the following experimental conditions were chosen

$$\delta^2 = 1.9,\ \varepsilon = 0.036,\ \tau = 3.5\cdot10^{-6}\ \text{sec};$$
$$\delta^2 = 30,\ \varepsilon = 0.027,\ \tau = 10^{-6}\ \text{sec},$$

where $\varepsilon = 2Q/CU_0^2$ is the ratio of the energy required for phase transformation to the energy stored in the capacitor; $\tau \approx \sqrt{LC}$, current risetime in the circuit; C, capacitance; U_0, capacitor charge voltage; and L, circuit inductance.

The electrical energy was stored in a capacitor with $C = 2.6\cdot10^{-6}$ F, and a mechanical switch was used. All experiments were performed with copper foil.

In processing the experimental data, the singular-point search algorithm was used [11], permitting an increase in measurement accuracy.

Figure 4 presents the dependence of resistance on energy introduced for various values of δ^2 (curves 1-4).

Curve 1 was calculated by Eq. (3), i.e., assuming infinite thermal conductivity (or very slow introduction of energy). Comparison of the calculated function with experiment for this case is difficult, since with slow heating, instability develops in the foil, leading to its destruction. Curve 4 corresponds to the converse limiting case $\delta^2 \to \infty$. Comparison with experiment is also difficult here, since at a very high rate of energy introduction it is necessary to consider nonuniform current distribution over the conductor section due to skin effect.

For finite values of δ^2 the function R(E/Q) naturally lies within the region limited by curves 1 and 4; curves 2 and 3 were constructed from numerical solutions of system (8) for $\delta^2 = 1.9$ and 30.† Figure 4 also shows experimental data for these values of δ^2 (points 5, 6), averaged over a series of ten experiments. The measurement error in the experiments was less than 5%.

The following principles follow from examination of the functions R(E/Q) obtained by calculation and experiment.

1. After the beginning of fusion the energy supplied may be expended in melting of the foil material, superheating of the solid phase, and heating of the liquid phase above T_f. The basic contribution to resistance increase is produced by the energy spent in melting the foil material, since the conductivity of the liquid phase $\sigma_2 \sim 0.5\sigma_1$. Therefore, for small values of δ^2 the function R(E/Q) is steeper than for larger δ^2.

2. For $\delta^2 \gg 1$, because of heating of the liquid phase above T_f, at the end of the process the melted foil is on the average heated to a temperature $T > T_f$. Therefore, its resistance will exceed that which corresponds to T_f in the liquid phase. This fact has been confirmed both by computation and experiment.

3. For both $\delta^2 = 1.9$ and $\delta^2 = 30$, the calculated functions R(E/Q) agree quantitatively with experimental values R*(E/Q). However, the qualitative behavior of R differs somewhat from that of R*.

For almost the entire process R < R*. In our opinion, this deviation is not caused by the simplifying assumptions (n = const, etc.). Apparently this is a consequence of instability of the planar fusion front – at certain points the foil is melted across its entire width. The liquid inclusions thus developed may be responsible for shifting of the experimental curve to higher values than the calculated curve, to as much as $\sigma_2 \sim 0.5\ \sigma_1$.

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†For curve 3 the calculations could not be carried out to completion of the process because of the incorrectness of the difference scheme for large gradients $\partial T/\partial x$

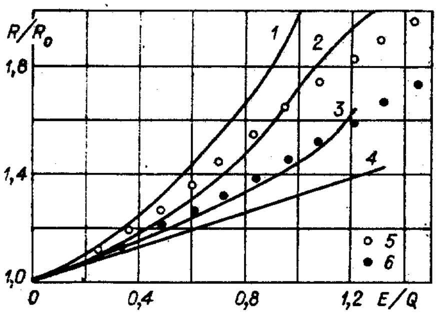

Fig. 4

Near the end of the process the foil may be regarded as a melt, in which "pieces" of solid phase are contained. For one and the same mass of solid phase the surface area S may prove to be larger than S*, so that fusion occurs more intensely for the unperturbed front and thus $R^* < R$ (Fig. 4). We might consider calculating the conductivity of such a system in a manner similar to that used in solid state physics ("flow theory") for strongly inhomogeneous media [12]. However, that problem is beyond the scope of the present study.

It should be noted that the assumption of a surface-type melting becomes invalid at large superheating levels. It can be shown from the results of [13] that for heating beyond the melting point by about 10°, intense formation of liquid-phase nuclei within the conductor volume commences. However, for sufficiently homogeneous specimens and rapid energy introduction into the conductor the dimensions of these nuclei will most probably be small, producing a small contribution from volume fusion.

Thus, compairson of calculations and experimental data shows that the surface melting model describes the initial stage, fusion, of electrical explosion of conductors with sufficient accuracy, both quantitatively and qualitatively.

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